Congruence Exercise 16.1

Question: 1

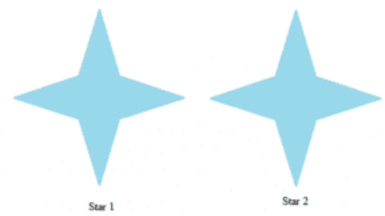
Explain the concept of congruence of figures with the help of certain examples.

Solution:

Congruent objects or figures are exact copies of each other or we can say mirror images of each other. The relation of two objects being congruent is called congruence.

Consider Ball A and Ball B. These two balls are congruent.

Now consider the two stars below. Star A and Star B are exactly the same in size, colour and shape. These are congruent stars.



Question: 2

Fill in the blanks:

(i) Two line segments are congruent if ___

(ii) Two angles are congruent if ___

(iii) Two square are congruent if ___

(iv) Two rectangles are congruent if ___

(v) Two circles are congruent if ___

Solution:

(i) They have the same length, since they can superpose on each other.

(ii) Their measures are the same. On superposition, we can see that the angles are equal.

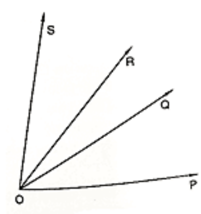
(iii) Their sides are equal. All the sides of a square are equal and if two squares have equal sides, then all their sides are of the same length. Also angles of a square are 90° which is also the same for both the squares.

(iv) Their lengths are equal and their breadths are also equal. The opposite sides of a rectangle are equal. So if two rectangles have lengths of the same size and breadths of the same size, then they are congruent to each other.

(v) Their radii are of the same length. Then the circles will have the same diameter and thus will be congruent to each other.

Question: 3

In Figure, $\angle POQ \cong \angle ROS$, can we say that $\angle POR \cong \angle QOS$



Solution:

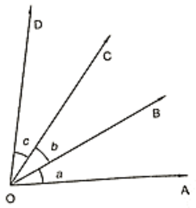
We have,

$\angle POQ \cong \angle ROS$ (1) Also, $\angle ROQ \cong \angle ROQ$ Therefore adding $\angle ROQ$ to both sides of (1), Weget, $\angle POQ + \angle ROQ \cong \angle ROQ + \angle R$

Question: 4

In figure, a = b = c, name the angle which is congruent to ∠AOC

Solution:



We have,

∠ AOB = ∠ BOC = ∠ COD

Therefore, ∠ AOB = ∠ COD

Also, ∠ AOB + ∠ BOC = ∠ BOC + ∠ COD

∠ AOC = ∠ BOD

Hence, ∠ BOD is congruent to ∠ AOC

Question: 5

Is it correct to say that any two right angles are congruent? Give reasons to justify your answer.

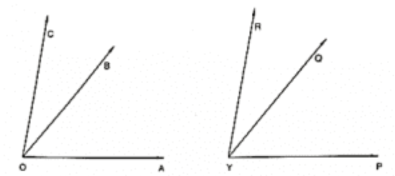
Solution:

Two right angles are congruent to each other because they both measure 90 degrees.

We know that two angles are congruent if they have the same measure.

Question: 6

In figure, ∠AOC ≅ ∠PYR and ∠BOC ≅ ∠QYR. Name the angle which is congruent to ∠AOB.



Solution:

∠AOC ≅ ∠PYR…. (i) Also, ∠BOC ≅ ∠QYR Now, ∠AOC = ∠AOB + ∠BOC ∠PYR = ∠PYQ +∠QYR By putting the value of ∠AC

Question: 7

Which of the following statements are true and which are false;

(i) All squares are congruent.

(ii) If two squares have equal areas, they are congruent.

(iii) If two rectangles have equal areas, they are congruent.

(iv) If two triangles have equal areas, they are congruent.

Solution:

(i) False.

All the sides of a square are of equal length.

However, different squares can have sides of different lengths. Hence all squares are not congruent.

(ii) True.

Area of a square = side x side

Therefore, two squares that have the same area will have sides of the same lengths. Hence they will be congruent.

(iii) False Area of a rectangle = length x breadth

Two rectangles can have the same area. However, the lengths of their sides can vary and hence they are not congruent.

Example: Suppose rectangle 1 has sides 8 m and 8 m and area 64 meter square. Rectangle 2 has sides 16 m and 4 m and area 64 meter square. Then rectangle 1 and 2 are not congruent.

(iv) False

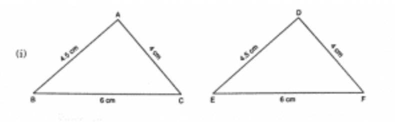
Area of a triangle = 12 x base x height

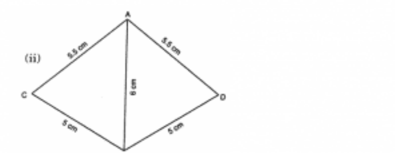
Two triangles can have the same area but the lengths of their sides can vary and hence they cannot be congruent.

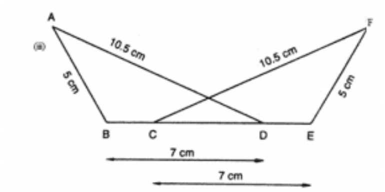
Congruence Exercise 16.2

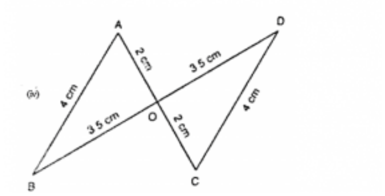
Question: 1

In the following pairs of triangle (Figures), the lengths of the sides are indicated along sides. By applying SSS condition, determine which are congruent. State the result in symbolic.

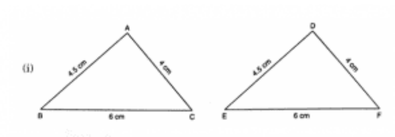








Solution:



1) In Δ ABC and Δ DEF

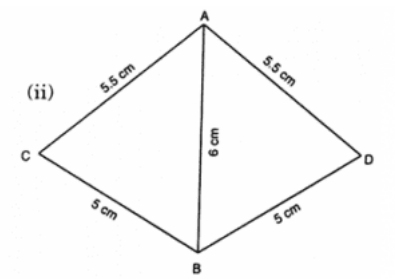
AB = DE = 4.5 cm (Side)

BC = EF = 6 cm (Side) and

AC = DF = 4 cm (Side)

Therefore, by SSS criterion of congruence, ΔABC ≅ ΔDEF

2)



In Δ ACB and Δ ADB

AC = AD (Side)

BC = BD (Side) and

AB = AB (Side)

Therefore, by SSS criterion of congruence, ΔACB ≅ ΔADB

3) In Δ ABD and Δ FEC,

AB = FE (Side)

AD = FC (Side)

BD = CE (Side)

Therefore, by SSS criterion of congruence, ΔABD ≅ ΔFEC

4) In Δ ABO and Δ DOC,

AB = DC (Side)

AO = OC (Side)

BO = OD (Side)

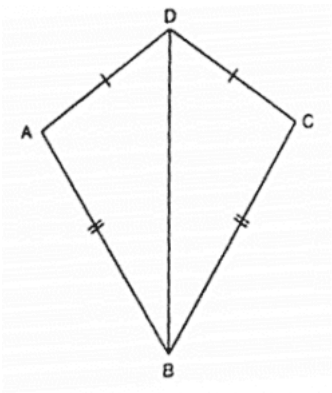
Therefore, by SSS criterion of congruence, ΔABO ≅ ΔODC

Question: 2

In figure, AD = DC and AB = BC

(i) Is ΔABD ≅ ΔCBD?

(ii) State the three parts of matching pairs you have used to answer (i).



Solution:

Yes ΔABD = ΔCBD by the SSS criterion. We have used the three conditions in the SSS criterion as follows:

AD = DC

AB = BC and

DB = BD

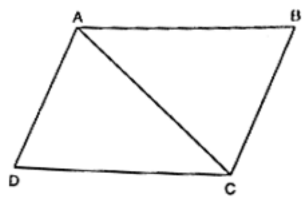
Question: 3

In Figure, AB = DC and BC = AD.

(i) Is ΔABC ≅ ΔCDA?

(ii) What congruence condition have you used?

(iii) You have used some fact, not given in the question, what is that?



Solution:

We have AB = DC

BC = AD

and AC = AC

Therefore by SSS ΔABC ≅ ΔCDA

We have used Side congruence condition with one side common in both the triangles.

Yes, have used the fact that AC = CA.

Question: 4

In ΔPQR ≅ ΔEFD,

(i) Which side of ΔPQR equals ED?

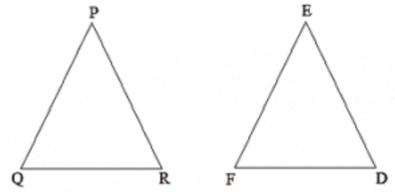
(ii) Which angle of ΔPQR equals angle E?

Solution:

ΔPQR ≅ ΔEFD

(i) Therefore PR = ED since the corresponding sides of congruent triangles are equal.

(ii) ∠QPR = ∠FED since the corresponding angles of congruent triangles are equal.



Question: 5

Triangles ABC and PQR are both isosceles with AB = AC and PO = PR respectively. If also, AB = PQ and BC = QR, are the two triangles congruent? Which condition do you use?

It ∠B = 50°, what is the measure of ∠R?

Solution:

We have AB = AC in isosceles ΔABC

And PQ = PR in isosceles ΔPQR.

Also, we are given that AB = PQ and QR = BC.

Therefore, AC = PR (AB = AC, PQ = PR and AB = PQ)

Hence, ΔABC ≅ ΔPQR

Now

∠ABC = ∠PQR (Since triangles are congruent)However, ΔPQR is isosceles.

Therefore, ∠PRQ = ∠PQR = ∠ABC = 50°

Question: 6

ABC and DBC are both isosceles triangles on a common base BC such that A and D lie on the same side of BC. Are triangles ADB and ADC congruent? Which condition do you use? If ∠

Solution:

∠BAD = ∠CAD (c.p.c.t)

∠BAD + ∠CAD = 40°/ 2 ∠BAD = 40°

∠BAD = 40°/2 =20°

∠ABC + ∠BCA + ∠BAC = 180° (Angle sum property)

Since ΔABC is an isosceles triangle,

∠ABC = ∠BCA ∠ABC +∠ABC + 40°= 180°

2 ∠ABC = 180° – 40° = 140° ∠ABC = 140°/2 = 70°

∠DBC + ∠ BCD + ∠ BDC = 180° (Angle sum property)

Since ΔABC is an isosceles triangle, ∠ DBC = ∠BCD ∠DBC + ∠DBC + 100^{o} = 180°

2 ∠DBC = 180° – 100^{o} = 80°

∠DBC = 80°/2 = 40°

In Δ BAD,

∠ABD + ∠BAD + ∠ADB = 180°(Angle sum property)

30° + 20° + ∠ADB = 180° (∠ADB = ∠ABC – ∠DBC), ∠ADB = 180°- 20° – 30°

∠ADB = 130°

∠ADB =130°

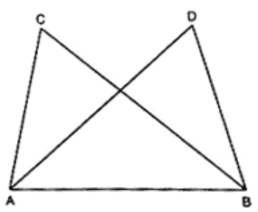
Question: 7

Δ ABC and ΔABD are on a common base AB, and AC = BD and BC = AD as shown in Figure. Which of the following statements is true?

(i) ΔABC ≅ ΔABD

(ii) ΔABC ≅ ΔADB

(iii) ΔABC ≅ ΔBAD



Solution:

In ΔABC and ΔBAD we have,

AC = BD (given)

BC = AD (given)

and AB = BA (common)

Therefore by SSS criterion of congruency, ΔABC ≅ ΔBAD

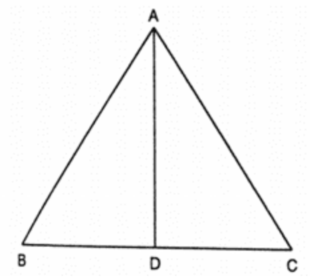
There option (iii) is true.

Question: 8

In Figure, ΔABC is isosceles with AB = AC, D is the mid-point of base BC.

(i) Is ΔADB ≅ ΔADC?

(ii) State the three pairs of matching parts you use to arrive at your answer.



Solution:

We have AB = AC.

Also since D is the midpoint of BC, BD = DC

Also, AD = DA

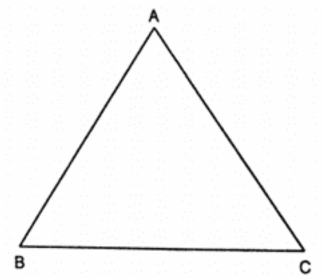
Therefore by SSS condition,

ΔADB ≅ ΔADC

We have used AB, AC : BD, DC AND AD, DA

Question: 9

In figure, ΔABC is isosceles with AB = AC. State if ΔABC ≅ ΔACB. If yes, state three relations that you use to arrive at your answer.



Solution:

Yes, ΔABC ≅ ΔACB by SSS condition.

Since, ABC is an isosceles triangle, AB = BC, BC = CB and AC = AB

Question: 10

Triangles ABC and DBC have side BC common, AB = BD and AC = CD. Are the two triangles congruent? State in symbolic form, which congruence do you use? Does ∠ABD equal ∠ACD? Why or why not?

Solution:

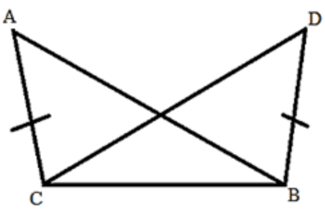
Yes,

Given,

Δ ABC and Δ DBC have side BC common, AB = BD and AC = CD

By SSS criterion of congruency, ΔABC ≅ ΔDBC

No, ∠ABD and ∠ACD are not equal because AB ≠ AC



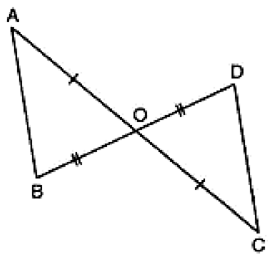
Congruence Exercise 16.3

Question: 1

By applying SAS congruence condition, state which of the following pairs of triangle are congruent. State the result in symbolic form

Solution:

(i)

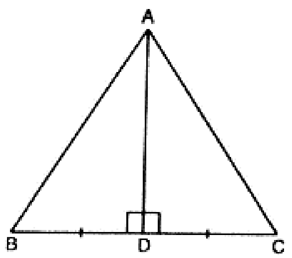


(i)

We have OA = OC and OB = OD and

∠AOB = ∠COD which are vertically opposite angles. Therefore by SAS condition, ΔAOC ≅ ΔBOD

(ii)



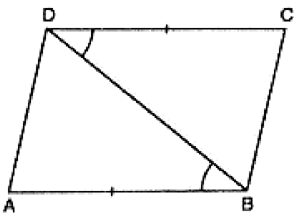
(ii)

We have BD = DC

∠ADB = ∠ADC = 90° and

Therefore, by SAS condition, ΔADB ≅ ΔADC.

(iii)



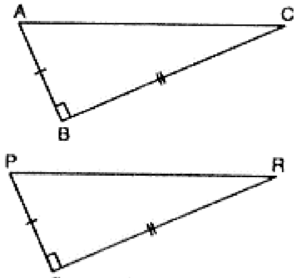
(iii)

We have AB = DC

∠ABD = ∠CDB and

Therefore, by SAS condition, ΔABD ≅ ΔCBD

(iv)



(iv)

We have BC = QR

ABC = PQR = 90°

And AB = PQ

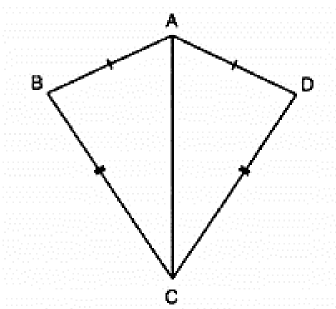
Therefore, by SAS condition, ΔABC≅ ΔPQR.

Question: 2

State the condition by which the following pairs of triangles are congruent.

Solution:

(i)



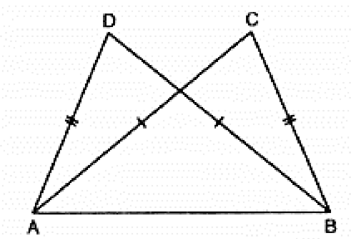
(i)

AB = AD

BC = CD and AC = CA

Therefore by SSS condition, ΔABC≅ ΔADC

(ii)



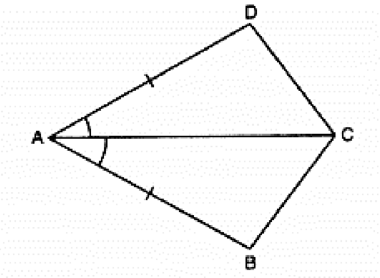
(ii)

AC = BD

AD = BC and AB = BA

Therefore, by SSS condition, ΔABD ≅ ΔADC

(iii)



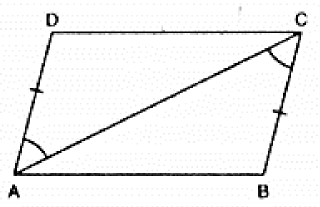
(iii)

AB = AD

∠BAC = ∠DAC and

Therefore by SAS condition, ΔBAC ≅ ΔBAC

(iv)



(iv)

AD = BC

∠DAC = ∠BCA and

Therefore, by SAS condition, ΔABC ≅ ΔADC

Question: 3

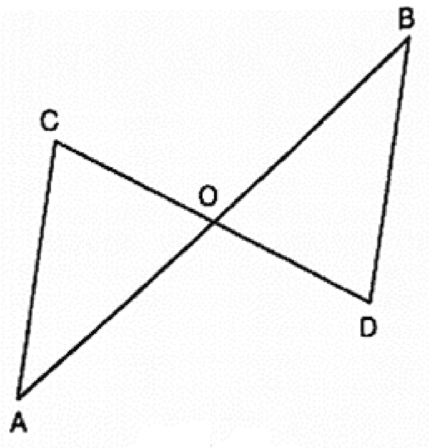
In figure, line segments AB and CD bisect each other at O. Which of the following statements is true?

(i) ΔAOC ≅ ΔDOB

(ii) ΔAOC ≅ ΔBOD

(iii) ΔAOC ≅ ΔODB

State the three pairs of matching parts, you have used to arrive at the answer.



Solution:

We have,

And, CO = OD

Also, AOC = BOD

Therefore, by SAS condition, ΔAOC ≅ ΔBOD

Question: 4

Line-segments AB and CD bisect each other at O. AC and BD are joined forming triangles AOC and BOD. State the three equality relations between the parts of the two triangles that are given or otherwise known. Are the two triangles congruent? State in symbolic form, which congruence condition do you use?

Solution:

We have AO = OB and CO = OD since AB and CD bisect each other at 0.

Also ∠AOC = ∠BOD since they are opposite angles on the same vertex.

Therefore by SAS congruence condition, ΔAOC ≅ ΔBOD

Question: 5

ΔABC is isosceles with AB = AC. Line segment AD bisects ∠A and meets the base BC in D.

(i) Is ΔADB ≅ ΔADC?

(ii) State the three pairs of matching parts used to answer (i).

(iii) Is it true to say that BD = DC?

Solution:

(i) We have AB = AC (Given)

∠BAD = ∠CAD (AD bisects ∠BAC)

Therefore by SAS condition of congruence, ΔABD ≅ ΔACD

(ii) We have used AB, AC; ∠BAD = ∠CAD; AD, DA.

(iii) Now, ΔABD≅ΔACD therefore by c.p.c.t BD = DC.

Question: 6

In Figure, AB = AD and ∠BAC = ∠DAC.

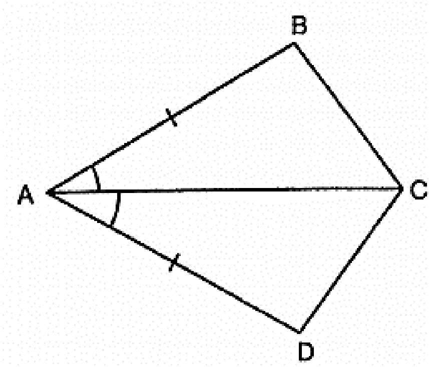
(i) State in symbolic form the congruence of two triangles ABC and ADC that is true.

(ii) Complete each of the following, so as to make it true:

(a) ∠ABC =

(b) ∠ACD =

(c) Line segment AC bisects ___ and ___



Solution:

i) AB = AD (given)

∠BAC = ∠DAC (given)

AC = CA (common)

Therefore by SAS condition of congruency, ΔABC ≅ ΔADC

ii) ∠ABC = ∠ADC (c.p.c.t)

∠ACD = ∠ACB (c.p.c.t)

Question: 7

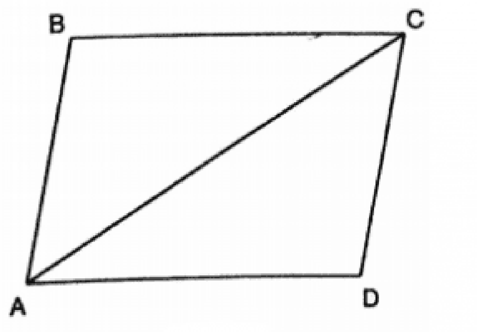
In figure, AB || DC and AB = DC.

(i) Is ΔACD ≅ ΔCAB?

(ii) State the three pairs of matching parts used to answer (i).

(iii) Which angle is equal to ∠CAD ?

(iv) Does it follow from (iii) that AD || BC?



Solution:

(i) Yes by SAS condition of congruency, ΔDCA ≅ ΔBAC

(ii) We have used AB = DC, AC = CA and ∠DCA = ∠BAC.

(iii) ∠CAD = ∠ACB since the two triangles are congruent.

(iv) Yes this follows from AD // BC as alternate angles are equal. If alternate angles are equal the lines are parallel

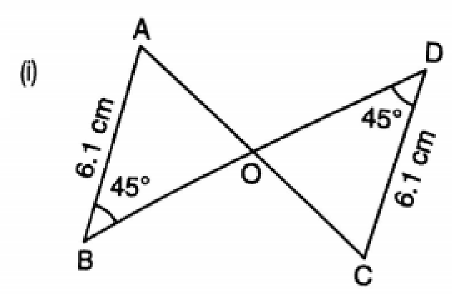
Congruence Exercise 16.4

Question: 1

Which of the following pairs of triangle are congruent by ASA condition?

Solution:

i)



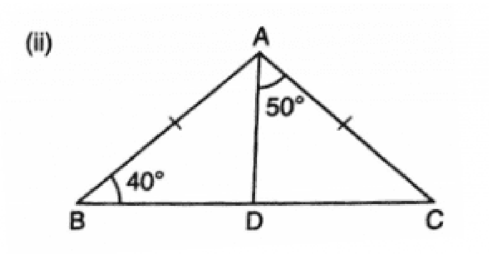
We have,

Since ∠ABO = ∠CDO = 45° and both are alternate angles, AB // DC, ∠BAO = ∠DCO (alternate angle, AB // CD and AC is a transversal line)

∠ABO = ∠CDO = 45° (given in the figure) Also, AB = DC (Given in the figure)

Therefore, by ASA ΔAOB ≅ ΔDOC

ii)



In ABC,

Now AB =AC (Given)

∠ABD = ∠ACD = 40° (Angles opposite to equal sides)

∠ABD + ∠ACD + ∠BAC = 180° (Angle sum property)

40° + 40° + ∠BAC=180°

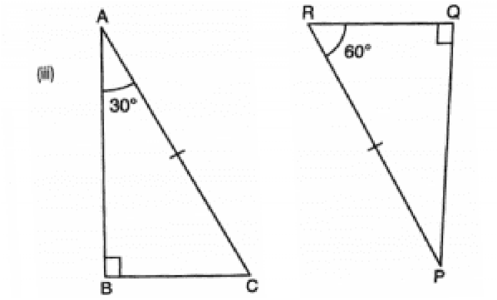
∠BAC =180°- 80° =100°

∠BAD + ∠DAC = ∠BAC ∠BAD = ∠BAC – ∠DAC = 100° – 50° = 50°

∠BAD = ∠CAD = 50°

Therefore, by ASA, ΔABD ≅ ΔADC

iii)



In Δ ABC,

∠A + ∠B + ∠C = 180°(Angle sum property)

∠C = 180°- ∠A – ∠B ∠C = 180° – 30° – 90° = 60°

In PQR,

∠P + ∠Q + ∠R = 180°(Angle sum property)

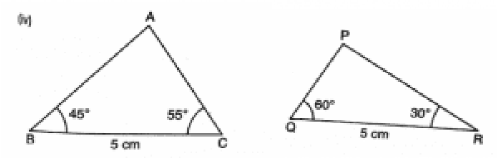
∠P = 180° – ∠Q – ∠R ∠P = 180°- 60°- 90° = 30°

∠BAC = ∠QPR = 30°

∠BCA = ∠PRQ = 60° and AC = PR (Given)

Therefore, by ASA, ΔABC ≅ ΔPQR

iv)



We have only

BC = QR but none of the angles of ΔABC and ΔPQR are equal.

Therefore, ΔABC and Cong ΔPRQ

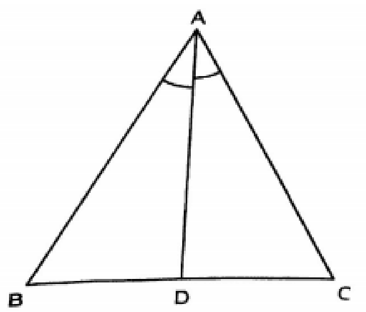
Question: 2

In figure, AD bisects A and AD and AD ⊥ BC.

(i) Is ΔADB ≅ ΔADC?

(ii) State the three pairs of matching parts you have used in (i)

(iii) Is it true to say that BD = DC?



Solution:

(i) Yes, ΔADB≅ΔADC, by ASA criterion of congruency.

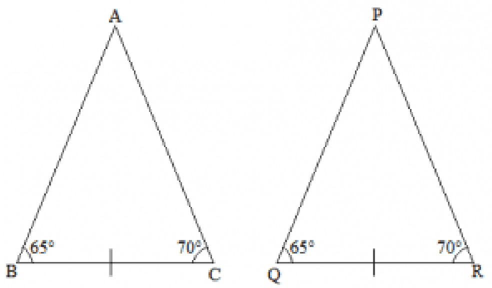
(ii) We have used ∠BAD = ∠CAD ∠ADB = ∠ADC = 90°

Since, AD ⊥ BC and AD = DA

(iii) Yes, BD = DC since, ΔADB ≅ ΔADC

Question: 3

Draw any triangle ABC. Use ASA condition to construct other triangle congruent to it.



Solution:

We have drawn

Δ ABC with ∠ABC = 65° and ∠ACB = 70°

We now construct ΔPQR ≅ ΔABC has ∠PQR = 65° and ∠PRQ = 70°

Also we construct ΔPQR such that BC = QR

Therefore by ASA the two triangles are congruent

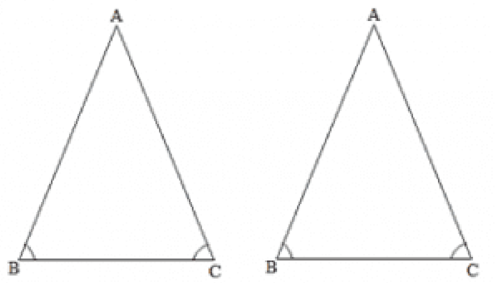
Question: 4

In Δ ABC, it is known that ∠B = C. Imagine you have another copy of Δ ABC

(i) Is ΔABC ≅ ΔACB

(ii) State the three pairs of matching parts you have used to answer (i).

(iii) Is it true to say that AB = AC?



Solution:

(i) Yes ΔABC ≅ ΔACB

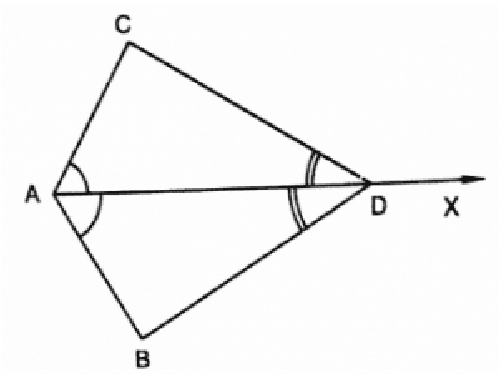
(ii) We have used ∠ABC = ∠ACB and ∠ACB = ∠ABC again.

Also BC = CB

(iii) Yes it is true to say that AB = AC since ∠ABC = ∠ACB.

Question: 5

In Figure, AX bisects ∠BAC as well as ∠BDC. State the three facts needed to ensure that ΔACD ≅ ΔABD



Solution:

As per the given conditions, ∠CAD = ∠BAD and ∠CDA = ∠BDA (because AX bisects ∠BAC)

AD = DA (common)

Therefore, by ASA, ΔACD ≅ ΔABD

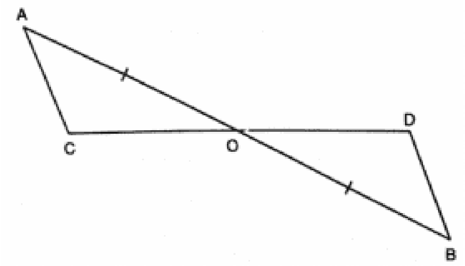
Question: 6

In Figure, AO = OB and ∠A = ∠B.

(i) Is ΔAOC ≅ ΔBOD

(ii) State the matching pair you have used, which is not given in the question.

(iii) Is it true to say that ∠ACO = ∠BDO?



Solution:

We have

∠OAC = ∠OBD,

AO = OB

Also, ∠AOC = ∠BOD (Opposite angles on same vertex)

Therefore, by ASA ΔAOC ≅ ΔBOD

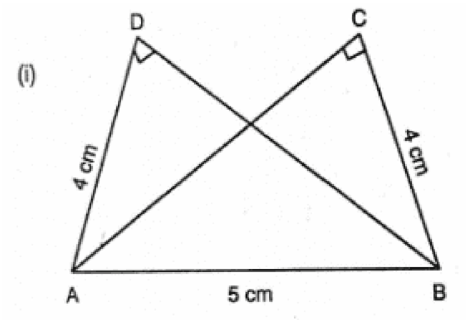
Congruence Exercise 16.5

Question: 1

In each of the following pairs of right triangles, the measures of some part are indicated alongside. State by the application of RHS congruence conditions which are congruent, and also state each result in symbolic form.

Solution:

i)

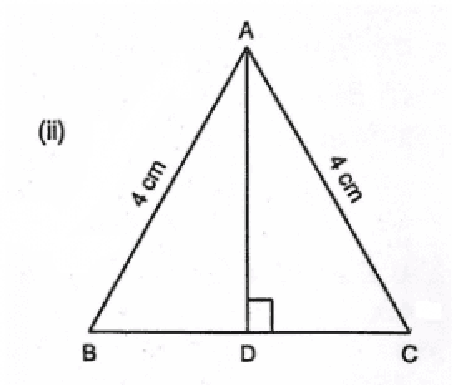


∠ADC = ∠BCA = 90°

AD = BC and hyp AB = hyp AB

Therefore, by RHS ΔADB ≅ ΔACB

ii)



AD = AD (Common)

hyp AC = hyp AB (Given)

∠ADB + ∠ADC = 180° (Linear pair)

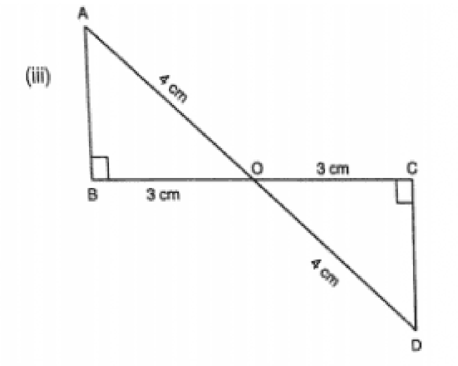
∠ADB + 90° = 180°

∠ADB = 180° – 90° = 90°

∠ADB = ∠ADC = 90°

Therefore, by RHS Δ ADB = Δ ADC

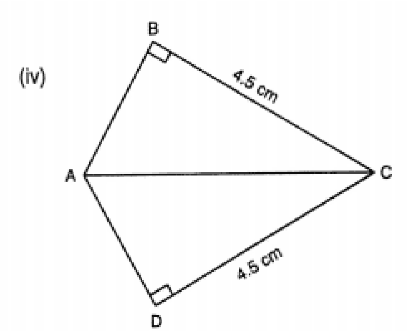
iii)



hyp AO = hyp DOBO = CO ∠B = ∠C = 90°

Therefore, by RHS, ΔAOB≅ΔDOC

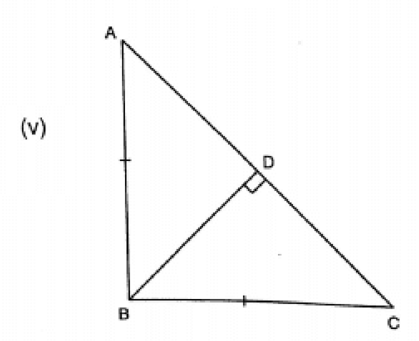
iv)



Hyp A = Hyp CABC = DC ∠ABC = ∠ADC = 90°

Therefore, by RHS, ΔABC ≅ ΔADC

v)



BD = DB Hyp AB = Hyp BC, as per the given figure,

∠BDA + ∠BDC = 180°

∠BDA + 90° = 180°

∠BDA= 180°- 90° = 90°

∠BDA = ∠BDC = 90°

Therefore, by RHS, ΔABD ≅ ΔCBD

Question: 2

Δ ABC is isosceles with AB = AC. AD is the altitude from A on BC.

i) Is ΔABD ≅ ΔACD?

(ii) State the pairs of matching parts you have used to answer (i).

(iii) Is it true to say that BD = DC?

Solution:

(i) Yes, ΔABD ≅ ΔACD by RHS congruence condition.

(ii) We have used Hyp AB = Hyp AC

AD = DA

∠ADB = ∠ADC = 90° (AD ⊥ BC at point D)

(iii)Yes, it is true to say that BD = DC (c.p.c.t) since we have already proved that the two triangles are congruent.

Question: 3

ΔABC is isosceles with AB = AC. Also. AD ⊥ BC meeting BC in D. Are the two triangles ABD and ACD congruent? State in symbolic form. Which congruence condition do you use? Which side of ADC equals BD? Which angle of Δ ADC equals ∠B?

Solution:

We have AB = AC ...... (i)

AD = DA (common) ......(ii)

And, ∠ADC = ∠ADB (AD ⊥ BC at point D) ......(iii)

Therefore, from (i), (ii) and (iii), by RHS congruence condition, ΔABD ≅ ΔACD, the triangles are congruent.

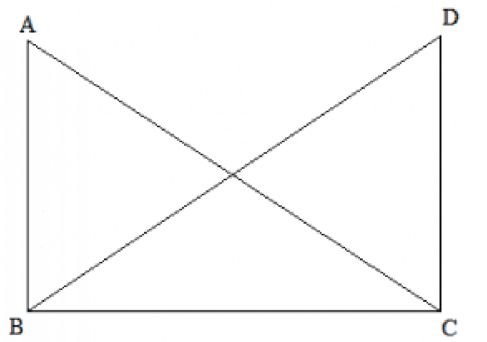
Therefore, BD = CD.

And ∠ABD = ∠ACD (c.p.c.t)

Question: 4

Draw a right triangle ABC. Use RHS condition to construct another triangle congruent to it.

Solution:



Consider

Δ ABC with ∠B as right angle.

We now construct another triangle on base BC, such that ∠C is a right angle and AB = DC

Also, BC = CB

Therefore, BC = CB

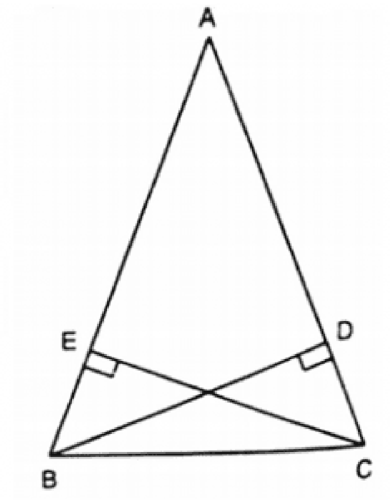
Therefore by RHS, ΔABC ≅ ΔDCB

Question: 5

In figure, BD and CE are altitudes of Δ ABC and BD = CE.

(i) Is ΔBCD ≅ ΔCBE?

(ii) State the three pairs or matching parts you have used to answer (i)



Solution:

(i) Yes, $\Delta BCD \cong \Delta CBE$ by RHS congruence condition.

(ii) We have used hyp BC = hyp CB

BD = CE (Given in question)

And $\angle BDC = \angle CBE = 90^{o}$