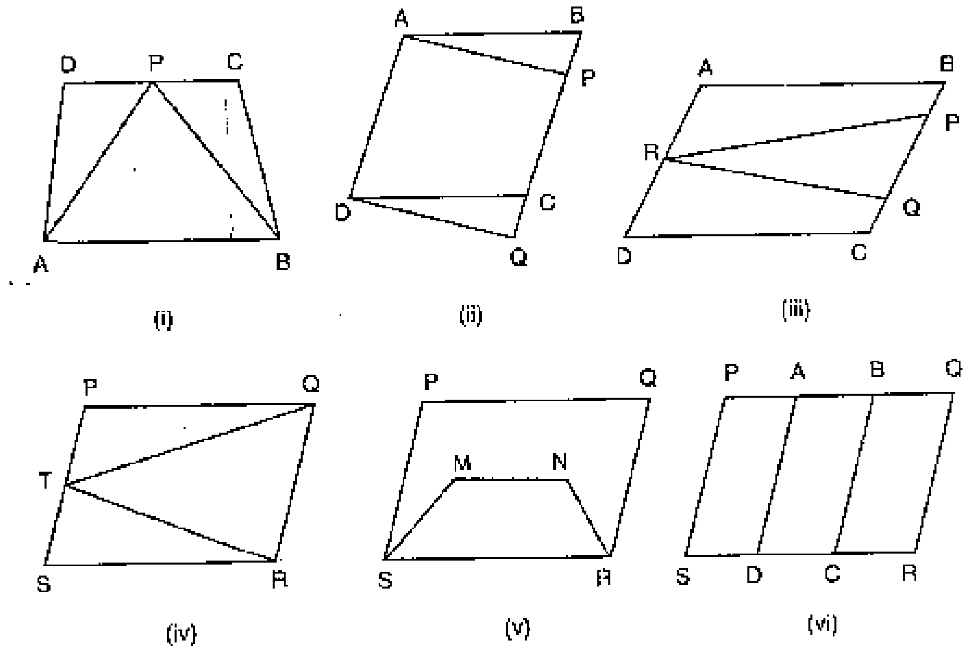
Exercise – 15.1

1. Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and two parallels.



Sol:

(i) ΔPCD and trapezium ABCD or on the same base CD and between the same parallels AB and DC.

(ii) Parallelogram ABCD and APQD are on the same base AD and between the same parallels AD and BQ.

(iii) Parallelogram ABCD and ΔPQR are between the same parallels AD and BC but they are not on the same base.

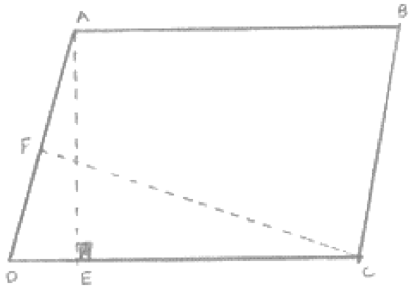
(iv) ΔQRT and parallelogram PQRS are on the same base QR and between the same parallels QR and PS

(v) Parallelogram PQRS and trapezium SMNR on the same base SR but they are not between the same parallels.

(vi) Parallelograms PQRS, AQRD, BCQR and between the same parallels also, parallelograms PQRS, BPSC and APSD are between the same parallels.

Exercise – 15.2

1. In fig below, ABCD is a parallelogram, AE ⊥ DC and CF ⊥ AD. If AB = 16 cm, AE = 8 cm and CF = 10 cm, find AD.



Sol:

Given that,

In a parallelogram $ABCD, CD = AB = 16cm$ [Opposite sides of a parallelogram are equal]

We know that,

Area of parallelogram = base $\times$corresponding attitude

Area of parallelogram $ABCD = CD \times AE = AD \times CF$

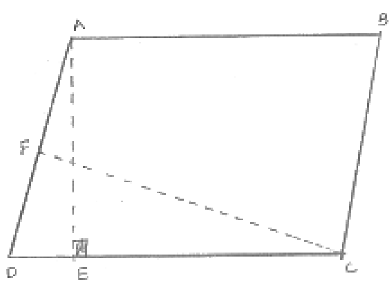
$16cm \times 8cm = AD \times 10cm$

$$AD = \frac{16 \times 8}{10} cm = 12 \cdot 8cm$$

Thus, the length of AD is $12 \cdot 8cm$

2. In Q. No 1, if AD = 6 cm, CF = 10 cm, and AE = 8cm, find AB.

Sol:



We know that,

Area of parallelogram ABCD = AD$\times$CF ……(1)

Again area of parallelogram $ABCD = DC \times AE$ ……(2)

Compare equation (1) and equation (2)

$AD \times CF = DC \times AE$

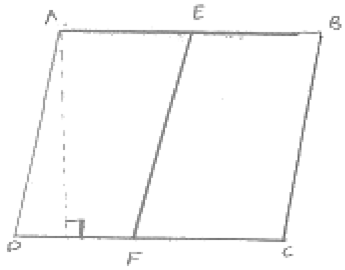
$\Rightarrow 6 \times 10 = D \times B$

$$\Rightarrow D = \frac{60}{8} = 7 \cdot 5cm$$

$\therefore AB = DC = 7 \cdot 5cm$ [$\therefore$ Opposite sides of a parallelogram are equal]

3. Let ABCD be a parallelogram of area 124 cm^2. If E and F are the mid-points of sides AB and CD respectively, then find the area of parallelogram AEFD.

Sol:



Given,

Area of parallelogram $ABCD = 124 cm^2$

Construction: draw $AP \perp DC$

Proof:

Area of parallelogram $AFED = DF \times AP$ ....(1)

And area of parallelogram $EBCF = FC \times AP$ ......(2)

And $DF = FC$ .....(3) [F is the midpoint of DC]

Compare equation (1), (2) and (3)

Area of parallelogram $AEFD =$ Area of parallelogram $EBCF$

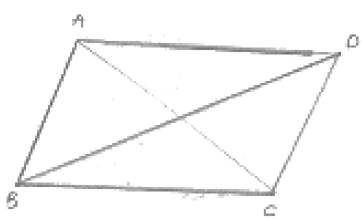
$\therefore$ Area of parallelogram $AEFD = \dfrac{\text{Area of parallelogram } ABCD}{2}$

$$= \frac{124}{2} = 62 cm^2$$

4. If ABCD is a parallelogram, then prove that

$ar\ (\Delta ABD) = ar\ (\Delta BCD) = ar\ (\Delta ABC) = ar\ (\Delta ACD) = \frac{1}{2} ar\ (||^{gm}\ ABCD)$

Sol:



Given: $ABCD$ is a parallelogram

To prove: area $(\Delta ABD) = ar(\Delta ABC) = are\ (\Delta ACD)$

$= \dfrac{1}{2} ar(||^{gm}\ ABCD)$

Proof: we know that diagonals of a parallelogram divides it into two equilaterals.

Since, AC is the diagonal.

Then, $ar(\Delta ABC) = ar(\Delta ACD) = \frac{1}{2} ar(\|^{gm}\ ABCD)$ ......(1)

Since, BD is the diagonal

Then, $ar(\Delta ABD) = ar(\Delta BCD) = \frac{1}{2} ar(\|^{gm}\ ABCD)$ .....(2)

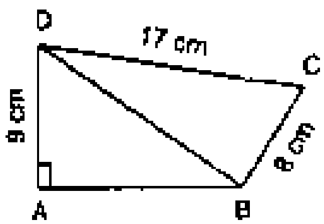
Compare equation (1) and (2)

$\therefore ar(\Delta ABC) = ar(\Delta ACD)$

$= ar(\Delta ABD) = ar(\Delta BCD) = \frac{1}{2} ar(\|^{gm}\ ABCD)$

Exercise – 15.3

1. In the below figure, compute the area of quadrilateral ABCD.



Sol:

Given that

$DC = 17cm$

$AD = 9cm$ and $BC = 8cm$

In ΔBCD we have

$CD^2 = BD^2 + BC^2$

$\Rightarrow (17)^2 = BD^2 + (8)^2$

$\Rightarrow BD^2 = 289 - 64$

$\Rightarrow BD = 15$

In ΔABD, we have

$AB^2 + AD^2 = BD^2$

$\Rightarrow (15)^2 = AB^2 + (9)^2$

$\Rightarrow AB^2 = 225 - 81 = 144$

$\Rightarrow AB = 12$

$ar(\text{quad, } ABCD) = ar(\Delta ABD) + ar(\Delta BCD)$

$\Rightarrow ar(\text{quad, } ABCD) = \frac{1}{2}(12 \times 9) + \frac{1}{2}(8 \times 17) = 54 + 68$

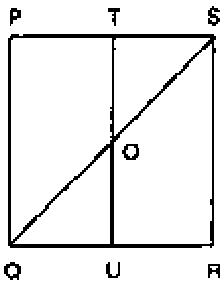
$= 112 cm^2$

$\Rightarrow ar$ (quad, $ABCD = \frac{1}{2}(12 \times 9) + \frac{1}{2}(8 \times 15)$

$= 54 + 60 cm^2$

$= 114 cm^2$

2. In the below figure, PQRS is a square and T and U are respectively, the mid-points of PS and QR. Find the area of ΔOTS if PQ = 8 cm.



Sol:

From the figure

T and U are the midpoints of PS and QR respectively.

$\therefore TU \parallel PQ$

$\Rightarrow TO \parallel PQ$

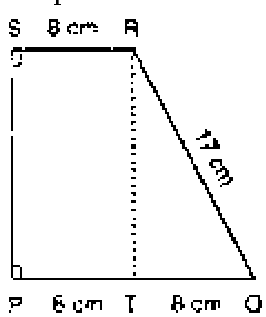
Thus, in $\Delta PQS, T$ is the midpoint of PS and $TO \parallel PQ$

$\therefore TO = \frac{1}{2} PQ = 4cm$

Also, $TS = \frac{1}{2} PS = 4cm$

$\therefore ar(\Delta OTS) = \frac{1}{2}(TO \times TS) = \frac{1}{2}(4 \times 4) cm^2 = 8cm^2$

3. Compute the area of trapezium PQRS is Fig. below.



Sol:

We have

$ar(\text{trap } PQRS) = ar(\text{rect } PSRT) + \text{are } a(\Delta QRT)$

$\Rightarrow ar(\text{trap}\cdot PQRS) = PT \times RT + \frac{1}{2}(QT \times RT)$

$= 8 \times RT + \frac{1}{2}(8 \times RT) = 12 \times RT$

In ΔQRT, we have

$QR^2 = QT^2 + RT^2$

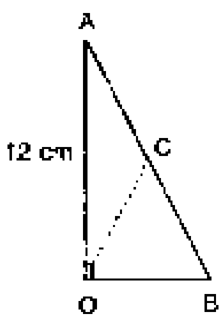
$\Rightarrow RT^2 = QR^2 - QT^2$

$\Rightarrow (RT)^2 = 17^2 - 8^2 = 225$

$\Rightarrow RT = 15$

Hence, $ar(\text{trap}\cdot PQRS) = 12 \times 15cm^2 = 180cm^2$

4. In the below fig. ∠AOB = 90°, AC = BC, OA = 12 cm and OC = 6.5 cm. Find the area of ΔAOB.



Sol:

Since, the midpoint of the hypotenuse of a right triangle is equidistant from the vertices

$\therefore CA = CB = OC$

$\Rightarrow CA = CB = 6\cdot 5cm$

$\Rightarrow AB = 13cm$

In a right angle triangle OAB, we have

$AB^2 = OB^2 + OA^2$

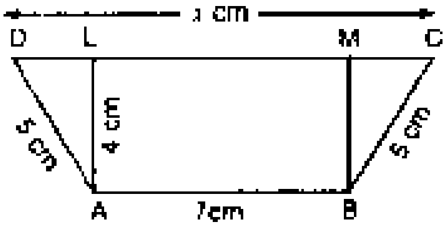
$\Rightarrow 13^2 = OB^2 + 12^2$

$\Rightarrow OB^2 = 13^2 - 12^2 = 169 - 144 = 25$

$\Rightarrow OB = 5$

$\therefore ar(\Delta AOB) = \frac{1}{2}(OA \times OB) = \frac{1}{2}(12 \times 5) = 30cm^2$

5. In the below fig. ABCD is a trapezium in which AB = 7 cm, AD = BC = 5 cm, DC = x cm, and distance between AB and DC is 4cm. Find the value of x and area of trapezium ABCD.



Sol:

Draw $AL \perp DC, BM \perp DC$ Then,

$AL = BM = 4cm$ and $LM = 7cm$

In ΔADL, we have

$AD^2 = AL^2 + DL^2 \Rightarrow 25 = 16 + DL^2 \Rightarrow DL = 3cm$

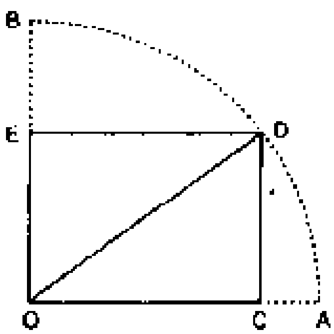
Similarly $MC = \sqrt{BC^2 - BM^2} = \sqrt{25-16} = 3cm$

$\therefore x = CD = CM + ML + CD = 3 + 7 + 3 = 13cm$

$ar(\text{trap} \cdot ABCD) = \frac{1}{2}(AB + CD) \times AL = \frac{1}{2}(7+13) \times 4cm^2$

$= 40cm^2$

6. In the below fig. OCDE is a rectangle inscribed in a quadrant of a circle of radius 10 cm. If OE = $2\sqrt{5}$, find the area of the rectangle.



Sol:

Given $OD = 10cm$ and $OE = 2\sqrt{5}cm$

By using Pythagoras theorem

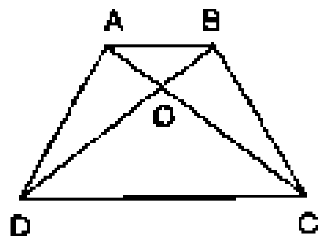
$\therefore OD^2 = OE^2 + DE^2$

$\Rightarrow DE = \sqrt{OD^2 - OF^2} = \sqrt{(10)^2 - (2\sqrt{5})^2} = 4\sqrt{5}cm$

$\therefore ar(\text{rect } DCDE) = OE \times DE = 2\sqrt{5} \times 4\sqrt{5}cm^2$

$= 40cm^2$ $\left[\because \sqrt{5} \times \sqrt{5} = 5\right]$

7. In the below fig. ABCD is a trapezium in which AB || DC. Prove that ar (ΔAOD) = ar(ΔBOC).



Sol:

Given: $ABCD$ is a trapezium with $AB \parallel DC$

To prove: $ar(\Delta AOD) = ar(BOC)$

Proof:

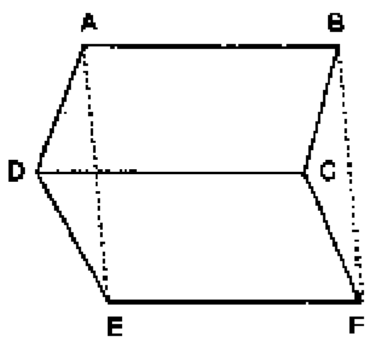
Since ΔADC and ΔBDC are on the same base DC and between same parallels AB and DC

Then, $ar(\Delta ADC = ar(\Delta BDC)$

$\Rightarrow ar(\Delta AOD) + ar(DOC) = ar(\Delta BOC) + ar(\Delta DOC)$

$\Rightarrow ar(\Delta AOD) = ar(\Delta BOC)$

8. In the given below fig. ABCD, ABFE and CDEF are parallelograms. Prove that ar (ΔADE) = ar (ΔBCF)



Sol:

Given that,

$ABCD$ is a parallelogram $\Rightarrow AD = BC$

$CDEF$ is a parallelogram $\Rightarrow DE = CF$

$ABFE$ is a parallelogram $\Rightarrow AE = BF$

Thus, in $\Delta s \ \ ADE \ and \ BCF$, we have

$AD = BC, DE = CF$ and $AE = BF$

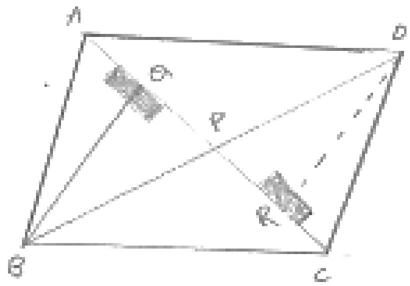
So, by SSS criterion of congruence, we have

$\Delta ADE \cong \Delta ABCF$

$\therefore ar(\Delta ADE) = ar(BCF)$

9. Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that:
ar(ΔAPB) × ar(ΔCPD) = ar(ΔAPD) × ar (ΔBPC)

Sol:



Construction: Draw $BQ \perp AC$ and $DR \perp AC$

Proof:

L.H.S

$= ar(\Delta APB) \times ar(\Delta CPD)$

$= \frac{1}{2}\left[\left(AP \times BQ\right)\right] \times \left(\frac{1}{2} \times PC \times DR\right)$

$= \left(\frac{1}{2} \times PC \times BQ\right) \times \left(\frac{1}{2} \times AP \times DR\right)$

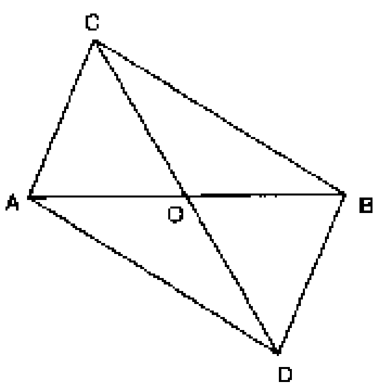
$= ar(\Delta BPC) \times ar(APD)$

$= RHS$

$\therefore LHS = RHS$

Hence proved.

10. In the below Fig, ABC and ABD are two triangles on the base AB. If line segment CD is bisected by AB at O, show that ar (Δ ABC) = ar (Δ ABD)



Sol:

Given that CD is bisected at O by AB

To prove: $ar(\Delta ABC) = ar(\Delta ABD)$

Construction: Draw CP $\perp$ AB and DQ $\perp$ AB

Proof:-

$ar(\Delta ABC) = \frac{1}{2} \times AB \times CP$ ....(i)

$ar(\Delta ABC) = \frac{1}{2} \times AB \times DQ$ ....(ii)

In $\angle CPO$ and ΔDQO

$\angle CPQ = \angle DQO$ $[Each\ 90°]$

Given that $CO = DO$

$\angle COP = \angle DOQ$ [vertically opposite angles are equal]

Then, $\Delta CPO \cong DQO$ [By AAS condition]

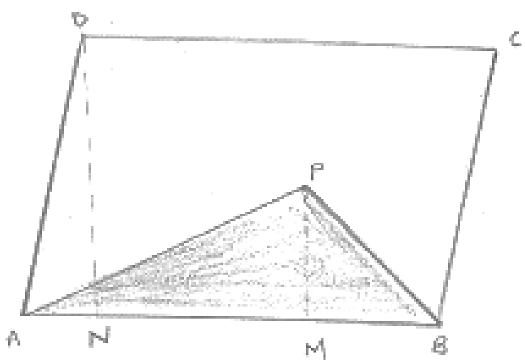
$\therefore CP = DQ$ ......(3) [CP.C.T]

Compare equation (1), (2) and (3)

Area $(\Delta ABC) = area\ of\ \ \Delta ABD$

11. If P is any point in the interior of a parallelogram ABCD, then prove that area of the triangle APB is less than half the area of parallelogram.

Sol:



Draw $DN \perp AB$ and $PM \perp AB$.

Now,

Area $(||^{gm}\ ABCD) = AB \times DN, ar(\Delta APB) = \frac{1}{2}(AB \times PM)$

Now, $PM < DN$

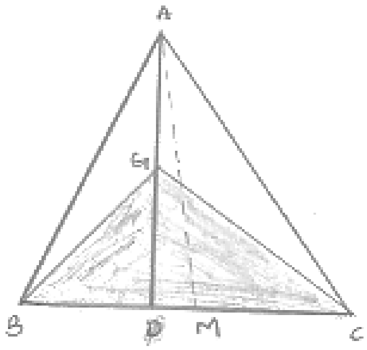
$\Rightarrow AB \times PM < AB \times DN$

$\Rightarrow \frac{1}{2}(AB \times PM) < \frac{1}{2}(AB \times DN)$

$\Rightarrow area(\Delta APB) < \frac{1}{2} ar(\text{Parragram } ABCD)$

12. If AD is a median of a triangle ABC, then prove that triangles ADB and ADC are equal in area. If G is the mid-point of median AD, prove that ar (Δ BGC) = 2 ar (Δ AGC).

Sol:



Draw $AM \perp BC$

Since, AD is the median of ΔABC

$\therefore BD = DC$

$\Rightarrow BD = AM = DC \times AM$

$\Rightarrow \frac{1}{2}(BD \times AM) = \frac{1}{2}(DC \times AM)$

$\Rightarrow ar(\Delta ABD) = ar(\Delta ACD)$ ....(i)

In $\Delta BGC, GD$ is the median

$\therefore ar(BGD) = area(OGD)$ ....(ii)

In $\Delta ACD, CG$ is the median

$\therefore$ area $(AGC) = area(\Delta CGD)$ ....(iii)

From (i) and (ii), we have

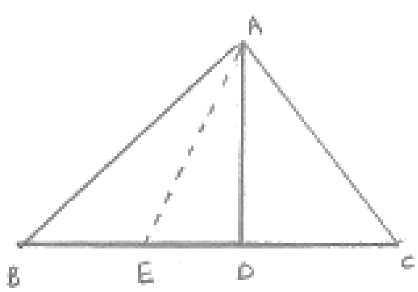
Area $(\Delta BGD) = ar(\Delta AGC)$

But, $ar(\Delta BGC) = 2ar(BGD)$

$\therefore ar(BGC) = 2ar(\Delta AGC)$

13. A point D is taken on the side BC of a ΔABC such that BD = 2DC. Prove that ar(Δ ABD) = 2ar (ΔADC).

Sol:



Given that,
In $\Delta ABC, BD = 2DC$

To prove: $ar(\Delta ABD) = 2ar(\Delta ADC)$

Construction: Take a point E on BD such that $BE = ED$
Proof: Since, $BE = ED$ and $BD = 2DC$
Then, $BE = ED = DC$
We know that median of Δ^{le} divides it into two equal Δ^{les}
$\therefore$ In $\Delta ABD, AE$ is a median

Then, area $(\Delta ABD) = 2ar(\Delta AED)$ .....(i)

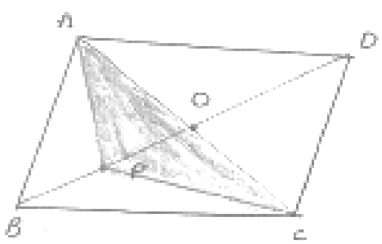
In $\Delta AEC, AD$ is a median

Then area $(\Delta AED) = area(\Delta ADC)$ .....(ii)

Compare equation (i) and (ii)
Area $(\Delta ABD) = 2ar(\Delta ADC)$.

14. ABCD is a parallelogram whose diagonals intersect at O. If P is any point on BO, prove that: (i) ar (ΔADO) = ar (ΔCDO) (ii) ar (ΔABP) = ar (ΔCBP)

Sol:



Given that $ABCD$ is a parallelogram

To prove: (i) $ar(\Delta ADO) = ar(\Delta CDO)$

(ii) $ar(\Delta ABP) = ar(\Delta CBP)$

Proof: We know that, diagonals of a parallelogram bisect each other
$\therefore AO = OC$ and $BO = OD$

(i) In ΔDAC, since DO is a median

Then area $(\Delta ADO) = area(\Delta CDO)$

(ii) In ΔBAC, Since BO is a median

Then; area $(\Delta BAO) = area(\Delta BCO)$ ......(1)

In a ΔPAC, Since PO is a median

Then, area $(\Delta PAO) = area(\Delta PCO)$ ......(2)

Subtract equation (2) from equation (1)

$\Rightarrow area(\Delta BAO) - ar(\Delta PAO) = ar(\Delta BCO) - area(\Delta PCO)$

$\Rightarrow Area(\Delta ABP) = Area\ of\ \ \Delta CBP$

15. ABCD is a parallelogram in which BC is produced to E such that CE = BC. AE intersects CD at F.

(i) Prove that ar (ΔADF) = ar (ΔECF)

(ii) If the area of ΔDFB = 3 cm^2, find the area of ||^{gm} ABCD.

Sol:

In triangles ADF and ECF, we have

$\angle ADF = \angle ECF$ [Alternative interior angles, Since $AD \| BE$]

$AD = EC$ [Since $AD = BC = CE$]

And $\angle DFA = \angle CFA$ [vertically opposite angles]

So, by AAS congruence criterion, we have

$\Delta ADF \cong ECF$

$\Rightarrow area(\Delta ADF) = area(\Delta ECF)\ \ and\ \ DF = CF.$

Now, $DF = CF$

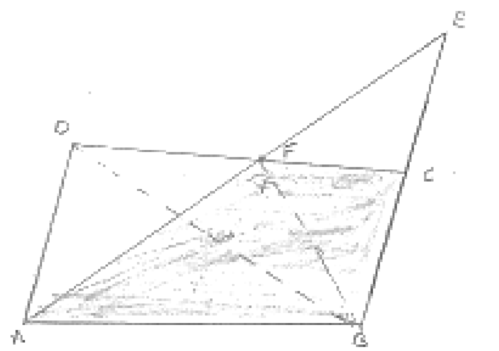
$\Rightarrow BF$ is a median in ΔBCD

$\Rightarrow area(\Delta BCD) = 2ar(\Delta BDF)$

$\Rightarrow area(\Delta BCD) = 2\times 3cm^2 = 6cm^2$

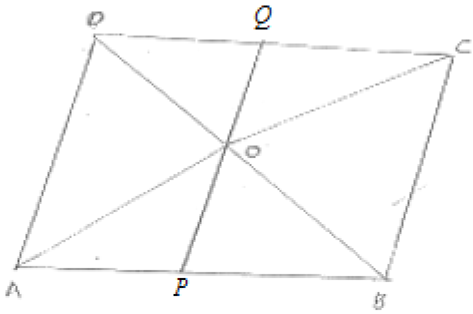
Hence, $ar(\|^{gm}\ ABCD) = 2ar(\Delta BCD) = 2\times 6cm^2$

$= 12cm^2$



16. ABCD is a parallelogram whose diagonals AC and BD intersect at O. A line through O intersects AB at P and DC at Q. Prove that ar (Δ POA) = ar (Δ QOC).

Sol:



In triangles POA and QOC, we have

$\angle AOP = \angle COQ$ [vertically opposite angles]

$OA = OC$ [Diagonals of a parallelogram bisect each other]

$\angle PAC = \angle QCA$ [$AB \parallel DC$; alternative angles]

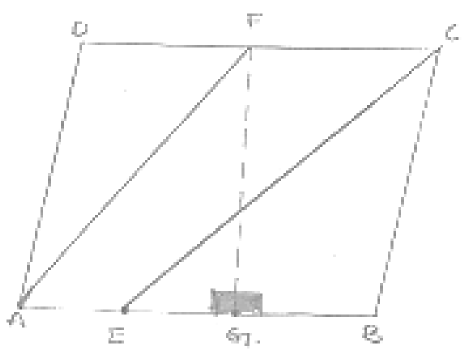
So, by ASA congruence criterion, we have

$\Delta POA \cong QOC$

Area $\left(\Delta POA\right) = area\left(\Delta QOC\right)$.

17. ABCD is a parallelogram. E is a point on BA such that BE = 2 EA and F is a point on DC such that DF = 2 FC. Prove that AE CF is a parallelogram whose area is one third of the area of parallelogram ABCD.

Sol:



Construction: Draw $FG \perp AB$

Proof: We have

$BE = 2EA$ and $DF = 2FC$

$\Rightarrow AB - AE = 2EA$ and $DC - FC = 2FC$

$\Rightarrow AB = 3EA$ and $DC = 3FC$

$\Rightarrow AE = \frac{1}{3}AB$ and $FC = \frac{1}{3}DC$ ......(1)

But $AB = DC$

Then, AE = DC [opposite sides of $||^{gm}$]

Then, $AE = FC$.

Thus, AE = FC and AE || FC.
Then, $AECF$ is a parallelogram
Now $ar\left(||^{gm}\ AECF\right) = AE \times FG$

$\Rightarrow ar\left(||^{gm}\ AECF\right) = \frac{1}{3} AB \times FG$ from (1)

$\Rightarrow 3ar\left(||^{gm}\ AECF\right) = AB \times FG$ ....(2)

and $area\left[||^{gm}\ ABCD\right] = AB \times FG$ ....(3)

Compare equation (2) and (3)

$\Rightarrow 3\ ar\left(||^{gm}\ AECF\right) = area\left(||^{gm}\ ABCD\right)$

$\Rightarrow area\left(||^{gm}\ AECF\right) = \frac{1}{3} area\left(||^{gm}\ ABCD\right)$

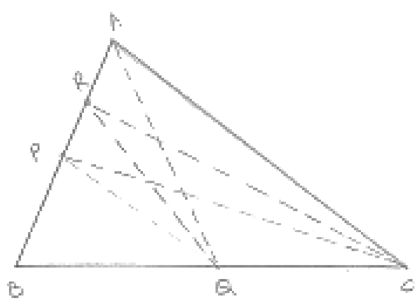
18. In a ΔABC, P and Q are respectively the mid-points of AB and BC and R is the mid-point of AP. Prove that :

(i) ar (Δ PBQ) = ar (Δ ARC)

(ii) ar (Δ PRQ) = $\frac{1}{2}$ ar (Δ ARC)

(iii) ar (Δ RQC) = $\frac{3}{8}$ ar (Δ ABC).

Sol:



(i) We know that each median of a Δ^{le} divides it into two triangles of equal area

Since, OR is a median of ΔCAP

$\therefore ar\left(\Delta CRA\right) = \frac{1}{2} ar\left(\Delta CAP\right)$ .....(i)

Also, CP is a median of ΔCAB

$\therefore ar\left(\Delta CAP\right) = ar\left(\Delta CPB\right)$ .....(ii)

From (i) and (ii) we get

$\therefore area\left(\Delta ARC\right) = \frac{1}{2} ar\left(CPB\right)$ ....(iii)

PQ is the median of ΔPBC

$\therefore area\left(\Delta CPB\right) = 2area\left(\Delta PBQ\right)$ .....(iv)

From (iii) and (iv) we get

$\therefore area(\Delta ARC) = area(PBQ)$ ......(v)

(ii) Since QP and QR medians of $\Delta^s QAB$ and QAP respectively.

$\therefore ar(\Delta QAP) = area(\Delta QBP)$ ....(vi)

And area $(\Delta QAP) = 2ar(\Delta QRP)$ ....(vii)

From (vi) and (vii) we have

Area $(\Delta PRQ) = \frac{1}{2} ar(\Delta PBQ)$ ....$(viii)$

From (v) and (viii) we get

Area $(\Delta PRQ) = \frac{1}{2} area(\Delta ARC)$

(iii) Since, $\angle R$ is a median of ΔCAP

$\therefore area(\Delta ARC) = \frac{1}{2} ar(\Delta CAP)$

$= \frac{1}{2} \times \frac{1}{2} \cdot ar(ABC)\Big]$

$= \frac{1}{4} area(ABC)$

Since RQ is a median of ΔRBC

$\therefore ar(\Delta RQC) = \frac{1}{2} ar(\Delta RBC)$

$= \frac{1}{2}\left[ar(\Delta ABC) - ar(ARC)\right]$

$= \frac{1}{2}\left[ar(\Delta ABC) - \frac{1}{4}(\Delta ABC)\right]$

$= \frac{3}{8}(\Delta ABC)$

19. ABCD is a parallelogram, G is the point on AB such that AG = 2 GB, E is a point of DC such that CE = 2DE and F is the point of BC such that BF = 2FC. Prove that:

(i) $ar(ADEG) = ar(GBCE)$

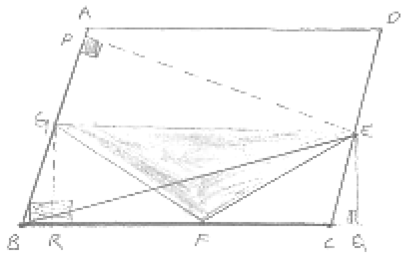
(ii) $ar(\Delta EGB) = \frac{1}{6}\ ar(ABCD)$

(iii) $ar(\Delta EFC) = \frac{1}{2} ar(\Delta EBF)$

(iv) $ar(\Delta EBG) = ar(\Delta EFC)$

(v) Find what portion of the area of parallelogram is the area of ΔEFG.

Sol:



Given,

$ABCD$ is a parallelogram

$AG = 2GB, CE = 2DE \text{ and } BF = 2FC$

To prove:

(i) $ar(ADEG) = ar(GBCE)$

(ii) $ar(\Delta EGB) = \frac{1}{6} are(ABCD)$

(iii) $ar(\Delta EFC) = \frac{1}{2} area(\Delta EBF)$

(iv) area $(\Delta EBG) = \frac{3}{2} area(EFC)$

(v) Find what portion of the area of parallelogram is the area of ΔEFG.

Construction: draw $EP \perp AB$ and $EQ \perp BC$

Proof : we have,

$AG = 2GB$ and $CE = 2DE$ and $BF = 2FC$

$\Rightarrow AB - GB = 2GB$ and $CD - DE = 2DE$ and $BC - FC = 2FC$

$\Rightarrow AB - GB = 2GB$ and $CD - DE = 2DE$ and $BC - FC = 2FC$.

$\Rightarrow AB = 3GB$ and $CD = 3DE$ and $BC = 3FC$

$\Rightarrow GB = \frac{1}{3} AB$ and $DE = \frac{1}{3} CD$ and $FC = \frac{1}{3} BC$ ....(i)

(i) $ar(ADEG) = \frac{1}{2}(AG + DE) \times EP$

$\Rightarrow ar(ADEG) = \frac{1}{2}\left(\frac{2}{3}AB + \frac{1}{3}CD\right) \times EP$ [By using (1)]

$\Rightarrow ar(ADEG) = \frac{1}{2}\left(\frac{2}{3}AB + \frac{1}{3}AB\right) \times EP$ $[\because AB = CD]$

$\Rightarrow ar(ADEG) = \frac{1}{2} \times AB \times EP$ .....(2)

And $ar(GBCE) = \frac{1}{2}(GB + CE) \times EP$

$$\Rightarrow ar(GBCE) = \frac{1}{2}\left[\frac{1}{3}AB + \frac{2}{3}CD\right] \times EP \qquad \text{[By using (1)]}$$

$$\Rightarrow ar(GBCE) = \frac{1}{2}\left[\frac{1}{3}AB + \frac{2}{3}AB\right] \times EP \qquad [\because AB = CD]$$

$$\Rightarrow ar(GBCE) = \frac{1}{2} \times AB \times EP \qquad .....(1)$$

Compare equation (2) and (3)

(ii) $ar(\Delta EGB) = \frac{1}{2} \times GB \times EP$

$= \frac{1}{6} \times AB \times EB$

$= \frac{1}{6} ar(1)^{9m}\ ABCD\Big].$

(iii) Area $(\Delta EFC) = \frac{1}{2} \times FC \times EQ \qquad ......(4)$

And area $(\Delta EBF) = \frac{1}{2} \times BF \times EQ$

$\Rightarrow ar(\Delta EBF) = \frac{1}{2} \times 2FC \times EQ \qquad [BF = 2FC\ \ given]$

$\Rightarrow ar(\Delta EBF) = FC \times EQ \qquad .....(5)$

Compare equation 4 and 5

Area $(\Delta EFC) = \frac{1}{2} \times area(\Delta EBF)$

(iv) From (i) part

$ar(\Delta EGB) = \frac{1}{6} ar(11^{5m}\ ABCD) \qquad .....(6)$

From (iii) part

$ar(\Delta EFC) = \frac{1}{2} ar(\Delta EBF)$

$\Rightarrow ar(\Delta EFC) = \frac{1}{3} ar(\Delta EBC)$

$\Rightarrow ar(\Delta EFC) = \frac{1}{3} \times \frac{1}{2} \times CE \times EP$

$= \frac{1}{2} \times \frac{1}{3} \times \frac{2}{3} CD \times EP$

$= \frac{1}{6} \times \frac{2}{3} \times ar(11^{gm}\ ABCD)$

$\Rightarrow ar(\Delta EFC) = \frac{2}{3} \times ar(\Delta EGB) \qquad \text{[By using]}$

$\Rightarrow ar(\Delta EGB) = \frac{3}{2} ar(EFC).$

(v) Area $(\Delta EFG) = ar(Trap \cdot BGEC) = -ar(\Delta BGF) \to (1)$

Now, area (trap BGEC) $= \frac{1}{2}(GB + EC) \times EP$

$= \frac{1}{2}\left(\frac{1}{3}AB + \frac{2}{3}CD\right) \times EP$

$= \frac{1}{2} AB \times EP$

$= \frac{1}{2} ar(11^{5m} ABCD)$

Area $(\Delta EFC) = \frac{1}{9} area(11^{5m} ABCD)$ [From iv part]

And area $(\Delta BGF) = \frac{1}{2} BF \times GR$

$= \frac{1}{2} \times \frac{2}{3} BC \times GR$

$= \frac{2}{3} \times \frac{1}{2} BC \times GR$

$= \frac{2}{3} \times ar(\Delta GBC)$

$= \frac{2}{3} \times \frac{1}{2} GB \times EP$

$= \frac{1}{3} \times \frac{1}{3} AB \times EP$

$= \frac{1}{9} AB \times EP$

$= \frac{1}{9} ar(11^{gm} ABCD)$ [From (1)]

$ar(\Delta EFG) = \frac{1}{2} ar(11^{gm} ABCD) = \frac{1}{9} ar(11^{gm} ABCD) = \frac{1}{9} ar(11^{gm} ABCD)$

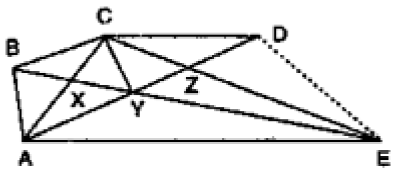
$= \frac{5}{18} ar(11^{gm} ABCD).$

20. In Fig. below, CD || AE and CY || BA.

(i) Name a triangle equal in area of ΔCBX

(ii) Prove that ar (Δ ZDE) = ar (ΔCZA)

(iii) Prove that ar (BCZY) = ar (Δ EDZ)



Sol:

Since, ΔBCA and ΔBYA are on the same base BA and between same parallels BA and CY

Then area $(\Delta BCA) = ar(BYA)$

$\Rightarrow ar(\Delta CBX) + ar(\Delta BXA) = ar(\Delta BXA) + ar(\Delta AXY)$

$\Rightarrow ar(\Delta CBX) = ar(\Delta AXY)$ ......(1)

Since, ΔACE and ΔADE are on the same base AE and between same parallels CD and AE

Then, $ar(\Delta ACE) = ar(\Delta ADE)$

$\Rightarrow ar(\Delta CLA) + ar(\Delta AZE) = ar(\Delta AZE) + ar(\Delta DZE)$

$\Rightarrow ar(\Delta CZA) = (\Delta DZE)$ ....(2)

Since ΔCBY and ΔCAY are on the same base CY and between same parallels BA and CY

Then $ar(\Delta CBY) = ar(\Delta CAY)$

Adding $ar(\Delta CYG)$ on both sides, we get

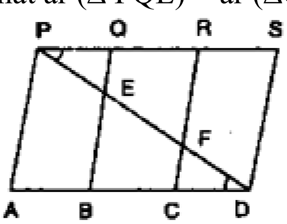
$\Rightarrow ar(\Delta CBX) + ar(\Delta CYZ) = ar(\Delta CAY) + ar(\Delta CYZ)$

$\Rightarrow ar(BCZX) = ar(\Delta CZA)$ ....(3)

Compare equation (2) and (3)

$ar(BCZY) = ar(\Delta DZE)$

21. In below fig., PSDA is a parallelogram in which PQ = QR = RS and AP || BQ || CR. Prove that ar (Δ PQE) = ar (ΔCFD).



Sol:

Given that PSDA is a parallelogram

Since, $AP \parallel BQ \parallel CR \parallel DS$ and $AD \parallel PS$

$\therefore PQ = CD$ .....(i)

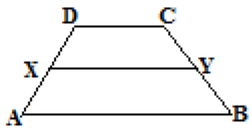
In ΔBED, C is the midpoint of BD and $CF \parallel BE$

$\therefore F$ is the midpoint of ED

$\Rightarrow EF = PE$
Similarly
$EF = PE$
$\therefore PE = FD$ .....(2)
In $\Delta SPQE$ and CFD, we have
$PE = FD$
$\angle EDQ = \angle FDC,$ [Alternative angles]
And $PQ = CD$
So by SAS congruence criterion, we have $\Delta PQE \cong \Delta DCF$.

22. In the below fig. ABCD is a trapezium in which AB || DC and DC = 40 cm and AB = 60 cm. If X and Y are respectively, the mid-points of AD and BC, prove that:
(i) XY = 50 cm
(ii) DCYX is a trapezium
(iii) ar (trap. DCYX) = $\frac{9}{11}$ ar (trap. (XYBA))

Sol:



(i) Join DY and produce it to meet AB produced at P
In $\Delta's$ BYP and CYD we have
$\angle BYP = (\angle CYD)$ [Vertical opposite angles]
$\angle DCY = \angle PBY$ $[\because DC \parallel AP]$
And $BY = CY$
So, by ASA congruence criterion, we have
$\Delta BYP \cong CYD$
$\Rightarrow DY = YP$ and $DC = BP$
$\Rightarrow y$ is the midpoint of DP
Also, x is the midpoint of AD
$\therefore XY \parallel AP$ and $XY = \frac{1}{2}AD$
$\Rightarrow xy = \frac{1}{2}(AB + BD)$
$\Rightarrow xy = \frac{1}{2}(BA + DC) \Rightarrow xy = \frac{1}{2}(60 + 40)$
(ii) We have
$XY \parallel AP$

$\Rightarrow XY \parallel AB \text{ and } AB \parallel DC$ [As proved above]

$\Rightarrow XY \parallel DC$

$\Rightarrow DCY$ is a trapezium

(iii) Since x and y are the midpoint of DA and CB respectively

$\therefore$ Trapezium DCXY and ABYX are of the same height say hm

Now

$$ar(Trap\ DCXY) = \frac{1}{2}(DC + XY) \times h$$

$$= \frac{1}{2}(50 + 40)hcm^2 = 45hcm^2$$

$$\Rightarrow ar(trap\ ABXY) = \frac{1}{2}(AB + XY) \times h = \frac{1}{2}(60 + 50)hm^3$$

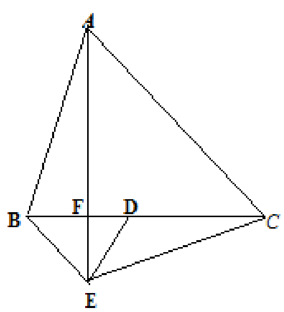
$$\Rightarrow ar(trap\ ABYX) = \frac{1}{2}(AB + XY) \times h = \frac{1}{2}(60 + 50)hcm^2$$

$$= 55cm^2$$

$$\frac{ar\ trap(YX)}{ar\ trap(ABYX)} = \frac{45h}{55h} = \frac{9}{11}$$

$$\Rightarrow ar(trap\ DCYX) = \frac{9}{11} ar(trap\ ABXY)$$

23. In Fig. below, ABC and BDE are two equilateral triangles such that D is the mid-point of BC. AE intersects BC in F. Prove that



(i) $ar(\Delta BDE) = \frac{1}{4} ar(\Delta ABC)$

(ii) $area(\Delta BDE) = \frac{1}{2} ar(\Delta BAE)$

(iii) $ar(BEF) = ar(\Delta AFD)$.

(iv) $area(\Delta ABC) = 2area(\Delta BEC)$

(v) $ar(\Delta FED) = \frac{1}{8} ar(\Delta AFC)$

(vi) $ar(\Delta BFE) = 2ar(\Delta EFD)$

Sol:

Given that,

ABC and BDE are two equilateral triangles.

Let $AB = BC = CA = x.$ Then $BD = \frac{x}{2} = DE = BE$

(i) We have

$$ar(\Delta ABC) = \frac{\sqrt{3}}{4}x^2$$

$$ar(\Delta ABC) = \frac{\sqrt{3}}{4}\left(\frac{x}{2}\right)^2 = \frac{1}{4}\times\frac{\sqrt{3}}{4}x^2$$

$$\Rightarrow ar(\Delta BDE) = \frac{\sqrt{3}}{4}\left(\frac{x}{2}\right)^2$$

(ii) It is given that triangles ABC and BED are equilateral triangles

$\angle ACB = \angle DBE = 60°$

$\Rightarrow BE \parallel AC$ (Since alternative angles are equal)

Triangles BAF and BEC are on the same base

BE and between the same parallel BE and AC

$\therefore ar(\Delta BAE) = area(\Delta BEC)$

$\Rightarrow ar(\Delta BAE) = 2ar(\Delta BDE)$

[$\because ED$ is a median of $\Delta EBC; ar(\Delta BEC) = 2ar(\Delta BDE)$]

$$\Rightarrow area(\Delta BDE) = \frac{1}{2}ar(\Delta BAE)$$

(iii) Since ΔABC and ΔBDE are equilateral triangles

$\therefore \angle ABC = 60°$ and $\angle BDE = 60°$

$\angle ABC = \angle BDE$

$\Rightarrow AB \parallel DE$ (Since alternative angles are equal)

Triangles BED and AED are on the same base ED and between the same parallels AB and DE.

$\therefore ar(\Delta BED) = area(\Delta AED)$

$\Rightarrow ar(\Delta BED) - area(\Delta EFD) = area(AED) - area(\Delta EFD)$

$\Rightarrow ar(BEF) = ar(\Delta AFD).$

(iv) Since ED is the median of ΔBEC

$\therefore area(\Delta BEC) = 2ar(\Delta BDE)$

$\Rightarrow ar(\Delta BEC) = 2\times\frac{1}{4}ar(\Delta ABC)$ [from (i)]

$\Rightarrow ar(\Delta BEC) = \frac{1}{2} area(\Delta ABC)$

$\Rightarrow area(\Delta ABC) = 2area(\Delta BEC)$

(v) Let h be the height of vertex E, corresponding to the side BD on triangle BDE
Let H be the height of the vertex A corresponding to the side BC in triangle ABC
From part (i)

$ar(\Delta BDE) = \frac{1}{4} ar(\Delta ABC)$

$\Rightarrow \frac{1}{2} \times BD \times h = \frac{1}{4} ar(\Delta ABC)$

$\Rightarrow BD \times h = \frac{1}{4}\left(\frac{1}{2} \times BC \times H\right)$

$\Rightarrow h = \frac{1}{2} H$ ....(1)

From part …..(iii)

Area $(\Delta BFE) = ar(\Delta AFD)$

$= \frac{1}{2} \times FD \times H$

$= \frac{1}{2} \times FD \times H$

$= 2\left(\frac{1}{2} \times FD \times 2h\right)$

$= 2ar(\Delta EFD)$

(vi) $area(\Delta AFC) = area(AFD) + area(ADC)$

$\Rightarrow ar(\Delta BFE) + \frac{1}{2} ar(\Delta ABC)$

[using part (iii); and AD is the median ΔABC]

$= ar(\Delta BFE) + \frac{1}{2} \times 4ar(\Delta BDE)$ using part (i)]

$= ar(\Delta BFE) = 2ar(\Delta FED)$ .....(3)

Area $(\Delta BDE) = ar(\Delta BFE) + ar(\Delta FED)$

$\Rightarrow R\ ar(\Delta FED) + ar(\Delta FED)$

$\Rightarrow 3\ ar(\Delta FED)$ ....(4)

From (2), (3) and (4) we get

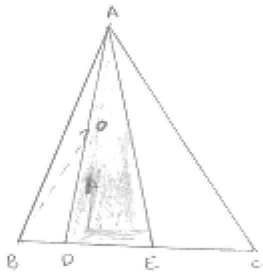
Area $(\Delta AFC) = 2area(\Delta FED) + 2 \times 3ar(\Delta FED)$

$= 8\ ar(\Delta FED)$

Hence, area $(\Delta FED) = \frac{1}{8} area(AFC)$

24. D is the mid-point of side BC of ΔABC and E is the mid-point of BD. if O is the mid-point of AE, prove that ar (ΔBOE) = $\frac{1}{8}$ ar (Δ ABC).

Sol:



Given that

D is the midpoint of side BC of ΔABC.

E is the midpoint of BD and

O is the midpoint of AE

Since AD and AE are the medians of $\Delta ABC\ and\ \Delta ABD$ respectively

$$\therefore ar(\Delta ABD) = \frac{1}{2} ar(\Delta ABC) \qquad ....(i)$$

$$ar(\Delta ABE) = \frac{1}{2} ar(\Delta ABD) \qquad ....(ii)$$

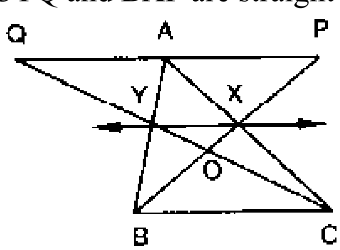
OB is a median of ΔABE

$$\therefore ar(\Delta BOE) = \frac{1}{2} ar(\Delta ABE)$$

From i, (ii) and (iii) we have

$$ar(BOE) = \frac{1}{8} ar(\Delta ABC)$$

25. In the below fig. X and Y are the mid-points of AC and AB respectively, QP || BC and CYQ and BXP are straight lines. Prove that ar (Δ ABP) = ar (ΔACQ).



Sol:

Since x and y are the midpoint AC and AB respectively

$\therefore XY \parallel BC$

Clearly, triangles $BYC\ and\ BXC$ are on the same base BC and between the same parallels $XY\ and\ BC$

$\therefore area(\Delta BYC) = area(BXC)$

$\Rightarrow area(\Delta BYC) = ar(\Delta BOC) = ar(\Delta BXC) - ar(BOC)$

$\Rightarrow ar(\Delta BOY) = ar(\Delta COX)$

$\Rightarrow ar(BOY) + ar(XOY) = ar(\Delta COX) + ar(\Delta XOY)$

$\Rightarrow ar(\Delta BXY) = ar(\Delta CXY)$

We observe that the quadrilateral XYAP and XYAQ are on the same base XY and between the same parallel XY and PQ.

$\therefore area(quad\ \ XYAP) = ar(quad\ \ XYPA) \qquad ....(ii)$

Adding (i) and (ii), we get

$ar(\Delta BXY) + ar(quad\ \ XYAP) = ar(CXY) + ar(quad\ \ XYQA)$

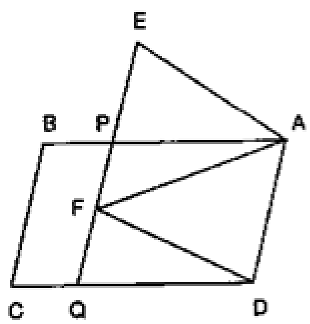
$\Rightarrow ar(\Delta ABP) = ar(\Delta ACQ)$

26. In the below fig. ABCD and AEFD are two parallelograms. Prove that

(i) PE = FQ

(ii) ar (Δ APE) : ar (ΔPFA) = ar Δ(QFD) : ar (Δ PFD)

(iii) $ar(\Delta PEA) = ar(\Delta QFD)$



Sol:

Given that, $ABCD\ and\ AEFD$ are two parallelograms

To prove: (i) $PE = FQ$

(ii) $\frac{ar(\Delta APE)}{ar(\Delta PFA)} = \frac{ar(\Delta QFD)}{ar(\Delta PFD)}$

(iii) $ar(\Delta PEA) = ar(\Delta QFD)$

Proof: (i) In $\Delta EPA\ and\ \Delta FQD$

$\angle PEA = \angle QFD$ [∵ Corresponding angles]

$\angle EPA = \angle FQD$ [Corresponding angles]

$PA = QD$ $\left[opp \cdot sides\ of\ 11^{gm}\right]$

Then, $\Delta EPA \cong \Delta FQD$ [By. AAS condition]

$\therefore EP = FQ \qquad [c.p.c.t]$

(ii) Since, $\Delta PEA\ and\ \Delta QFD$ stand on the same base PE and FQ lie between the same parallels EQ and AD

$\therefore ar(\Delta PEA) = ar(\Delta QFD) \to (1)$

$AD \qquad \therefore ar(\Delta PFA) = ar(PFD) \qquad .....(2)$

Divide the equation (i) by equation (2)

$$\frac{area\ of\ (\Delta PEA)}{area\ of\ (\Delta PFA)} = \frac{ar\Delta(QFD)}{ar\Delta(PFD)}$$

(iii) From (i) part $\Delta EPA \cong FQD$

Then, $ar(\Delta EDA) = ar(\Delta FQD)$

27. In the below figure, ABCD is parallelogram. O is any point on AC. PQ || AB and LM || AD. Prove that ar (||^{gm} DLOP) = ar (||^{gm} BMOQ)

Sol:

Since, a diagonal of a parallelogram divides it into two triangles of equal area

$\therefore area(\Delta ADC) = area(\Delta ABC)$

$\Rightarrow area(\Delta APO) + area(11^{gm}\ DLOP) + area(\Delta OLC)$

$\Rightarrow area(\Delta AOM) + ar(11gmDLOP) + area(\Delta OQC) \qquad .....(i)$

Since, AO and OC are diagonals of parallelograms AMOP and OQCL respectively.

$\therefore area(\Delta APO) = area(\Delta AMO) \qquad .....(ii)$

And, area $(\Delta OLC) = Area(\Delta OQC) \qquad ...(iii)$

Subtracting (ii) and (iii) from (i), we get

Area $(11^{gm}\ \ DLOP) = area(11^{gm}\ BMOQ)$

28. In a ΔABC, if L and M are points on AB and AC respectively such that LM || BC. Prove that:

(i) $ar(\Delta LCM) = ar(\Delta LBM)$

(ii) $ar\ (\Delta LBC) = ar(\Delta MBC)$

(iii) $ar(\Delta ABM) = ar(\Delta ACL)$

(iv) $ar(\Delta LOB) = ar(\Delta MOC)$

Sol:

(i) Clearly Triangles $LMB\ and\ LMC$ are on the same base LM and between the same parallels $LM\ \ and\ \ BC$.

$\therefore ar(\Delta LMB) = ar(\Delta LMC)$ ......(i)

(ii) We observe that triangles LBC and MBC area on the same base BC and between the same parallels LM and BC

$\therefore arc\ \Delta LBC = ar(MBC)$ .....(ii)

(iii) We have

$ar(\Delta LMB) = ar(\Delta LMC)$ [from (1)]

$\Rightarrow ar(\Delta ALM) + ar(\Delta LMB) = ar(\Delta ALM) + ar(LMC)$

$\Rightarrow ar(\Delta ABM) = ar(\Delta ACL)$

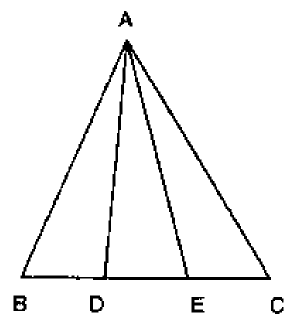
(iv) We have

$ar(\Delta CBC) = ar(\Delta MBC)$ $\therefore$ [from (1)]

$\Rightarrow ar(\Delta LBC) = ar(\Delta BOC) = a(\Delta MBC) - ar(BOC)$

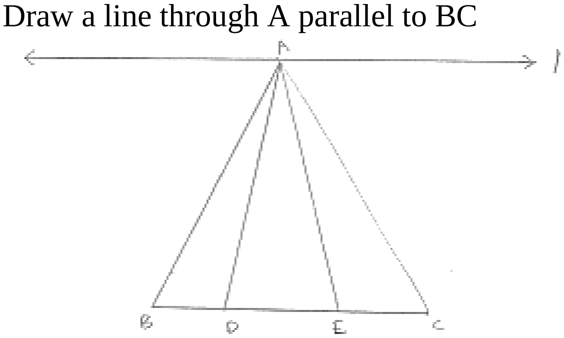
$\Rightarrow ar(\Delta LOB) = ar(\Delta MOC)$

29. In the below fig. D and E are two points on BC such that BD = DE = EC. Show that ar (ΔABD) = ar (ΔADE) = ar (ΔAEC).



Sol:

Draw a line through A parallel to BC



Given that, $BD = DE = EC$

We observe that the triangles ABD and AEC are on the equal bases and between the same parallels C and BC. Therefore, Their areas are equal.

Hence, $ar(ABD) = ar(\Delta ADE) = ar(\Delta ACDE)$

30. If below fig. ABC is a right triangle right angled at A, BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment AX ⊥ DE meets BC at Y. Show that:

(i) $\Delta MBC \cong \Delta ABD$

(ii) $ar\left(BYXD\right) = 2\,ar\left(\Delta MBC\right)$

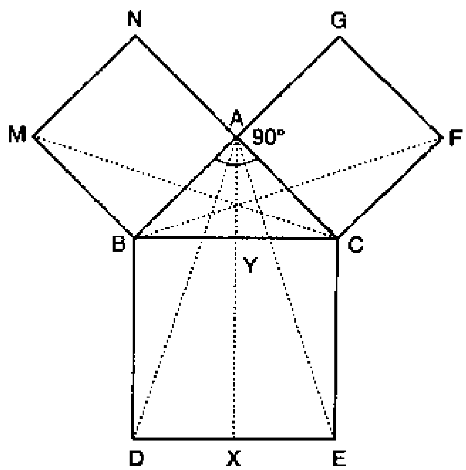
(iii) ar (BYXD) = ar (Δ ABMN)

(iv) ΔFCB ≅ Δ ACE

(v) ar (CYXE) = 2 ar (Δ FCB)

(vi) ar (CYXE) = ar (ACFG)

(vii) ar (BCED) = ar (ABMN) + ar (ACFG)



Sol:

(i) In ΔMBC and ΔABD, we have

$MB = AB$

$BC = BD$

And $\angle MBC = \angle ABD$

[$\because \angle MBC$ and $\angle ABC$ are obtained by adding $\angle ABC$ to a right angle]

So, by SAS congruence criterion, We have

$\Delta MBC \cong \Delta ABD$

$\Rightarrow ar\left(\Delta MBC\right) = ar\left(\Delta ABD\right)$ ....(1)

(ii) Clearly, ΔABC and $BYXD$ are on the same base BD and between the same parallels AX and BD

$$\therefore Area\left(\Delta ABD\right) = \frac{1}{2} Area\left(rect\ BYXD\right)$$

$$\Rightarrow ar\left(rect \cdot BYXD\right) = 2ar\left(\Delta ABD\right)$$

$$\Rightarrow are\left(rect \cdot BYXD\right) = 2ar\left(\Delta MBC\right) \qquad ......(2)$$

$$\left[\because ar\left(\Delta ABD\right) = ar\left(\Delta MBC\right) \qquad ......from\,(i)\right.$$

(iii) Since triangle $M \cdot BC$ and square $MBAN$ are on the same Base MB and between the same parallels MB and NC

$\therefore 2ar(\Delta MBC) = ar(MBAN)$ .....(3)

From (2) and (3) we have

$ar(sq \cdot MBAN) = ar(rect\ BYXD)$.

(iv) In triangles FCB and ACE we have

$FC = AC$

$CB = CF$

And $\angle FCB = \angle ACE$

[$\because \angle FCB\ and\ \angle ACE$ are obtained by adding $\angle ACB$ to a right angle]

So, by SAS congruence criterion, we have

$\Delta FCB \cong \Delta ACE$

(v) We have

$\Delta FCB \cong \Delta ACE$

$\Rightarrow ar(\Delta FCB) = ar(\Delta ECA)$

Clearly, ΔACE and rectangle CYXE are on the same base CE and between the same parallels CE and AX

$\therefore 2ar(\Delta ACE) = ar(CYXE)$ ....(4)

(vi) Clearly, ΔFCB and rectangle $FCAG$ are on the same base FC and between the same parallels FC and BG

$\therefore 2ar(\Delta FCB) = ar(FCAG)$ ......(5)

From (4) and (5), we get

Area $(CYXE) = ar(ACFG)$

(vii) Applying Pythagoras theorem in ΔACB, we have

$BC^2 = AB^2 + AC^2$

$\Rightarrow BC \times BD = AB \times MB + AC \times FC$

$\Rightarrow area(BCED) = area(ABMN) + ar(ACFG)$